



Back of the envelope estimates



Warning:

People get the right answers either

- (a) because they have done / looked up the detailed calculation and know the right answers
- (b) because they are really great scientists with a lot of intuition

Case (b) is relatively rare...

Topics

Preliminaries: Units, coordinates & other useful(?) stuff Gamma ray production by protons Basics Gamma-ray visibility of supernova remnants Seeing molecular clouds in gamma rays Radiation processes involving electrons Inverse Compton scattering Synchrotron radiation Radiation cooling Application to real objects Gamma ray propagation - will they reach us? Cosmic particle accelerators Supernova remnant kinematics Supernova remnant shocks Particle propagation and particle acceleration Some of the fine print The receiving end: Detecting gamma rays



Should do all of this on the blackboard, but

- (a) bit too time consuming
- (b) transparencies may help for reference

Stop me if I'm going too fast or something is unclear





Energy, distance

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1 erg = 10^{-7} J; 1 TeV \approx 1.6 erg;
Supernova E<sub>kin</sub> \approx 10<sup>51</sup> erg
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1 yr \approx \pi \ 10^7 \ s
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1 pc \approx 3.26 LJ \approx 3.1·10¹⁸ cm \approx 1000 km/s x 1000 yr Distance to center of Galaxy \approx 8.5 kpc Surface of kpc sphere \approx 1.2·10⁴⁴ cm² Distance to M31 (Andromeda) \approx 800 kpc Distance to Centaurus A \approx 4 Mpc

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Redshift z=0.1 \approx 0.4 Gpc
Surface of Gpc sphere \approx 1.2<sup>.</sup>10<sup>56</sup> cm<sup>2</sup>
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Gyroradius of (z=1) particles: r_{pc} \approx E_{PeV}/B_{\mu G}
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Instrument sensitivity

assume

100 detected photons for image (reasonable) background-free (somewhat optimistic)

optical telescope (few eV) 10 m aperture, 1 h, 100% eff $3 \cdot 10^{-8}$ ph/cm²s ~ 10^{-7} eV/cm²s ~ $2 \cdot 10^{-19}$ erg/cm²s X-ray satellite (keV) 50 ks, eff. mirror aperture 500 cm² (Chandra) 4.10^{-6} ph/cm²s ~ 10^{-14} erg/cm²s Fermi-LAT (few 100 MeV) required source 1 yr, 2 sr, eff. area 8000 cm² luminosity @ 1 kpc: $2 \cdot 10^{-9} \text{ ph/cm}^2 \text{s} \sim 10^{-12} \text{ erg/cm}^2 \text{s}$ $\sim 10^{32} \text{ erg/s}$ Cherenkov telescope (few 100 GeV) Sun thermal lumi 50 h, eff. area 50000 m² $\sim 4.10^{33}$ erg/s 10^{-12} ph/cm²s ~ 5.10⁻¹³ erg/cm²s Crab pulsar spin-down

 $\sim 5.10^{38} \text{ erg/s}$

Crab-like pulsar, assume 1% of spin-down (1% of 5.10³⁸ ergs/s) into radiation {note: actual Crab has 10⁻⁵ into VHE gamma rays)

at 1 kpc $4 \cdot 10^{-8} \text{ ergs/cm}^2 \text{s}$

at center of Galaxy

 5.10^{-10} ergs/cm²s

at LMC (50 kpc) 2.10^{-11} ergs/cm²s

at Andromeda

 $6 \cdot 10^{-14} \text{ ergs/cm}^2 \text{s}$

Dropping mass into BH, at 1% mc²

1 solar mass / yr @ 1 Gpc 5.10^{-12} ergs/cm²s



Mass, density

Solar mass $\approx 2.10^{30}$ kg Proton mass $\approx 1.7.10^{-27}$ kg

Typical interstellar density $\approx 1 \text{ H/cm}^3$ $\approx 5 \cdot 10^{28} \text{ kg/pc}^3$ $\approx 0.025 \text{ M}_{sol}/\text{pc}^3$ $\approx 1 \text{ M}_{sol}/\text{ sphere of 2 pc radius}$

Energy density

Typical interstellar magnetic field \approx few μ G \approx few 10^{-10} TEnergy density in cosmic rays $\approx 1 \text{ eV} / \text{ cm}^3$ Energy density in starlight $\approx 1 \text{ eV} / \text{ cm}^3$ Energy density in CMB $\approx 0.25 \text{ eV} / \text{ cm}^3$ Energy density in B-field $\approx 0.03 \text{ B}^2_{\mu\text{G}} \text{ eV} / \text{ cm}^3$ 1 eV / cm^3 $\approx 5 \cdot 10^{43} \text{ erg/pc}^3$



Particle (or photon) density and flux



usually use photon index, defined as

$$\Phi = \frac{d^3 N}{dA dE dt} = C E^{-\Gamma} = C' (E / E_0)^{-\Gamma}$$

for $E_o = 1$ TeV

- $C' \approx 3.5 \cdot 10^{-11} \text{ cm}^{-2} \text{s}^{-1} \text{TeV}^{-1}$
- $C' \approx 1.10^{-5} \text{ cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{TeV}^{-1}$

 $C' \approx 1.10^{-10} \text{ cm}^{-2} \text{s}^{-1} \text{TeV}^{-1}$

 $C' \approx 1.5 \cdot 10^{-8} \text{ cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{TeV}^{-1}$

 $C' \approx 1.5 \cdot 10^{-13} \text{ cm}^{-2} \text{s}^{-1} \text{TeV}^{-1}$

for Crab Nebula ($\Gamma \approx 2.5$) for cosmic ray protons ($\Gamma \approx 2.7$) ... within 0.1° radius for cosmic ray electrons ($\Gamma \approx 3$) ... within 0.1° radius

Crab Nebula with MAGIC CT



Spectral energy distribution (SED)

SED:
$$E^2 \frac{dN}{dE} = E \frac{dN}{d\log E}$$

~Energy flux per decade of spectrum

Units: Energy/time*area

also often written as

$$\nu F_{\nu}(\nu) = \nu \frac{dE}{d\nu}$$







Local coordinate system: alt, az



Celestial coordinates: RA, Dec



Celestial coordinates: RA, Dec



Galactic coordinates















Cross section for p H (i.e. p p) collisions:

Proton size ~ 1 fm Strong interactions have short range \rightarrow Cross section $\sigma O(\pi \ 10^{-30} \ m^2) \approx 30 \ mb$ (Reality $\approx 40 \ mb$)

Interaction rate $r = \sigma c \rho \approx 4 \cdot 10^{-26} cm^2 \ 3 \cdot 10^{10} cm/s n/cm^3$ $\approx 10^{-15} n/s \approx 3 \cdot 10^{-8} n/yr$ $\approx energy-independent!$





Gamma ray spectrum follows proton spectrum, roughly $\Phi_{\gamma}(E) \sim \Phi_{p}(10E)$, with features smeared over about one decade in energy, and appearing about a decade lower in energy

In particular, a power law proton spectrum $E^2 dN/dE$ gives a power law gamma spectrum with same index (± 0.2) **10⁻¹⁰** Gamma ray SED = Proton SED times 10⁻¹¹ cross section, convolved with $p \rightarrow gamma SED$ 10^{.12} Gammas from 100 TeV p 1 TeV p 10¹³ 10⁹ 10¹¹ 10

Valuable reference and parametrizations:

Energy spectra of gamma-rays, electrons and neutrinos produced at interactions of relativistic protons with low energy radiation

S.R. Kelner^{*} and F.A. Aharonian[†]

We derived simple analytical parametrizations for energy distributions of photons, electrons, and neutrinos produced in interactions of relativistic protons with an isotropic monochromatic radiation field. The results on photomeson processes are obtained using numerical simulations of protonphoton interactions based on the public available Monte-Carlo code SOPHIA. For calculations of energy spectra of electrons and positrons from the pair production (Bethe-Heitler) process we suggest a simple formalism based on the well-known differential cross-section of the process in the rest frame of the proton. The analytical presentations of energy distributions of photons and leptons provide a simple but accurate approach for calculations of broad-band energy spectra of gamma-rays and neutrinos in cosmic proton accelerators located in radiation dominated environments.

PACS numbers: 12.20.Ds, 13.20.Cz, 13.60.-r, 13.85.Qk

arXiv:0803.0688



Gamma ray visibility of SNR

Typical energy: 10^{51} erg; typical density $n=1/cm^3$ Shared between pdV work, heat, magnetic field, particles ... all are somehow related ... assume roughly equal shares ... $\rightarrow O(10\%) \approx 10^{50}$ erg in protons

Interaction rate O(10⁻¹⁵/s); 1/6 of energy into gammas \rightarrow gamma ray luminosity $\approx 2.10^{34}$ erg/s

Spread (~uniformly) over 7 decades of SED (~100 MeV (pion mass) to ~PeV) → TeV gamma ray luminosity ≈ 2.10³³ erg/s

Gamma ray energy flux $\Rightarrow \phi \approx 2.10^{33}$ (erg/s) $E_{51}n/(4\pi d^2) \approx 2.10^{-11} E_{51}n/d^2_{kpc}$ erg/(s cm²)

The "official" answer

Supernova remnant visibility in gamma rays Drury, Aharonian, Völk A&A 287 (1994) 959

 $\begin{array}{ll} \mathsf{F}(>1\mathsf{TeV}) & \approx 9 \cdot 10^{-11} \ \theta \ \mathsf{E}_{51} \ \mathsf{n}/\mathsf{d}^2_{\mathrm{kpc}} & \mathsf{ph/cm^2s} \\ & & \mathsf{I}_{\mathrm{Efficiency} \ \theta \ \approx \ 0.1} \\ & \approx 3.5 \cdot 10^{-11} \ \mathsf{E}_{51} \ \mathsf{n}/\mathsf{d}^2_{\mathrm{kpc}} & \mathrm{erg/cm^2s, \ 1...10 \ TeV} \end{array}$

Sensitivity limit: $\approx 10^{-12} \text{ erg/cm}^2 \text{s}$


Seeing molecular clouds in gamma rays



Seeing molecular clouds in gamma rays

Cosmic ray flux (p) $\phi \approx 10^{-1} \text{ E}^{-2.7}_{\text{TeV}}$ /TeV m² s srOther nuclei:x1.6Cosmic ray density $\rho_{CR} = 4\pi \phi/c$ Energy density0.0015 eV/cm³10-100 TeV

Correct answer:

Cosmic ray penetration in clouds

Cesarsky & Völk, A&A 70 (1978) 367 Everett & Zweibel, arXiv:1107.1243 Fatuzzo et al., arXiv:1010.0059 are they enhanced inside ? non-trivial if do they get in? curvature radius in B fields small compared to cloud size

→ discussion of CR propagation



... describes everything!



Inverse Compton Scattering



Bremsstrahlung



Synchrotron radiation





Exact treatment & detailed discussion



THE reference:

Bremsstrahlung, Synchrotron Radiation, and Compton Scattering of High-Energy Electrons Traversing Dilute Gases Blumenthal & Gould Reviews of Modern Physics, vol. 42, pp. 237-271



Cross section



total cross section σ depends only on cms energy \sqrt{s} in units where $\hbar = c = 1$, σ has dimension $1/E^2$ hence $\sigma \approx \alpha^2/s$ up to factors of order unity

Cross section

for simplicity, consider head-on collision

$$s = (E_e + E_{ph})^2 - (p_e - p_{ph})^2 \underset{E_e >> m_e}{\approx} m_e^2 + 4E_e E_{ph}$$

Thomson limit
$$4E_e E_{ph} << m_e^2$$

 $\rightarrow \wedge \wedge \wedge \wedge \wedge \wedge$

$$s \approx m_e^2$$
; $\sigma \approx \text{const} \approx \frac{\alpha^2}{m_e^2}$

Klein Nishina limit $4E_e E_{ph} >> m_e^2$

$$s \approx 4E_e E_{ph}$$
; $\sigma \approx \frac{\alpha^2}{4E_e E_{ph}}$

Cross section



Correct answer: $16 \rightarrow 6$ "Thomson cross section" $6.6 \cdot 10^{-29} \text{ m}^2$





Kinematics

CM frame m^* \swarrow $WWWWW E^*_{\gamma} = |p^*_{\gamma}| = |p^*_{e}|$

 $p_{e}^{*} << m_{e}$ (Thomson limit) $\Rightarrow E_{e}^{*} = \frac{p_{e}^{*2}}{2m_{e}} << |p_{e}^{*}| = |p_{\gamma}^{*}| = E_{\gamma}^{*}$

 \Rightarrow all energy carried by gamma

$$E *_{\gamma} \approx m * -m_e = \sqrt{m_e^2 + 4E_e E_{ph}} - m_e \approx \frac{2E_e E_{ph}}{m_e}$$

Lab frame: boost by $\gamma = \frac{E_e}{m^*} \approx \frac{E_e}{m_e} \implies E_{\gamma} \approx \frac{2E_e^2 E_{ph}}{m_e^2}$

In useful units

$$E_{\gamma}[TeV] = \frac{2E_{e}^{2}[TeV]E_{ph}[eV] \times 10^{12}}{\left(511 \times 10^{-9}\right)^{2}} \approx 8E_{e}^{2}[TeV]E_{ph}[eV]$$

Example: scattering off CMB, with $E_{ph} \approx 2 \times 10^{-4} \text{eV}$: $E_{\gamma}[TeV] \approx 0.002 E_e^2[TeV]$ 10 TeV electron off CMB: $E_{\gamma} = 0.2 \text{ TeV}$

What about energy conservation?? E_{γ} grows as E_e^2 !

Kinematics

 $E_{\gamma} \approx \frac{2E_e^2 E_{ph}}{m_e^2}$

in the Thomson limit where $4E_e E_{ph} << m_e^2$ and hence always $E_{\gamma} < E_e$!

In practical units: Thomson limit $E_e[TeV]E_{ph}[eV] < 0.1$ for scattering off visible light: $E_e < 100$ GeV for scattering off CMB: $E_e < 500$ TeV

Spectra of IC photons

$$E_{\gamma} \propto E_{e}^{2} E_{ph}$$

$$\frac{dN}{dE_{\gamma}} = \frac{dN}{dE_{e}} \frac{dE_{e}}{dE_{\gamma}} \propto E_{e}^{-\alpha} E_{\gamma}^{-1/2} \quad (\text{since } E_{e} \propto E_{\gamma}^{1/2})$$

$$\propto \left(E_{\gamma}^{1/2}\right)^{-\alpha} E_{\gamma}^{-1/2} = E_{\gamma}^{-(\alpha+1)/2}$$

$$\Rightarrow \text{ Gamma spectral index is } (\alpha+1)/2$$

$$E^{2} \frac{dN}{dE}$$
Gamma SED
$$E[ectron SED]$$

$$E_{\gamma,peak} \approx 8E_{e,max}^{2}[TeV]E_{ph}[eV]$$

Energy loss rate

 $E_{\gamma} \approx \frac{2E_e^2 E_{ph}}{m_e^2}$ and $\sigma \approx \frac{\alpha^2}{m_e^2}$

 $\frac{dE}{dt} = -E_{\gamma}\sigma nc \qquad \text{where } n = \text{density of target photons}$

$$n = \frac{U}{E_{ph}}; \quad U = \text{energy density of "target"}$$
$$\frac{dE}{dt} = -\left(\frac{2E_e^2 E_{ph}}{m_e^2}\right) \left(\frac{\alpha^2}{m_e^2}\right) \left(\frac{U}{E_{ph}}\right) c \propto E_e^2 U$$

Energy loss scales with square of electron energy depends only on energy density of target (B, vis, CMB, ...) field

Energy loss rate

Energy density in starlight

Energy density in CMB

Energy density in B-field

 $\tau = \frac{E_e}{(dE_e/dt)} \propto \frac{1}{E_e}$ $\tau \approx \frac{3 \cdot 10^5 \text{ yr}}{U[\text{eV/cm}^3] E[\text{TeV}]}$ $\tau_{sync} \approx \frac{10^7 \text{ yr}}{B^2[\mu\text{G}] E[\text{TeV}]}$

 \approx 1 eV / cm³

 \approx 0.26 eV / cm³

 $\approx 0.03 \text{ B}^2_{\mu\text{G}} \text{ eV} / \text{ cm}^3$

Energy density in 3 μ G field \approx CMB

for typical eV energy densities in target fields and multi-TeV electrons, loss time $O(10^5)$ y

IC scattering loss history

Blumenthal & Gould, RMP 42



FIG. 5. Sketch of a typical time evolution of an electron's energy due to losses by Compton scattering.



Synchrotron radiation



Start from E_{γ} [TeV] $\approx 8E_e^2$ [TeV] E_{ph} [eV] What is the photon energy ??

Synchrotron radiation energy

Start from E_{γ} [TeV] $\approx 8E_{e}^{2}$ [TeV] E_{ph} [TeV] What is the photon energy ??

Electron in B field has one characteristic frequency Gryo frequency $v = \frac{eB}{2\pi m_e} \approx 2.8B[\mu G]$ Hz Energy $E_{ph}[eV] \approx 10^{-14} B[\mu G]$ Sync. radiation $E_{sv}[eV] \approx 0.08E_e^2$ [TeV] $B[\mu G]$

Correct answer:



Electron spectrum with index 2 and (varied) cutoff E_{cutoff}

Shown: SED E²dN/dE

Electron spectrum is flat in this representation

Numerical integration of exact expressions incl. CMB spectrum











Electron spectrum with index 2 and (varied) cutoff E_{cutoff} IC strongly reduced in

> Klein-Nishina regime

$$4 \epsilon \gamma_e/m > 1$$

F. Moderski et al. astro-ph/0504388





Spectra & radiation cooling

Cooling modifies particle spectra

dE/dt = const (Ionization losses)

- → relative losses stronger for low-energy particles
- → spectra become harder

dE/dt ~ E (Bremsstrahlung, IC in KN regime)

- → relative losses same for all particles
- → (power-law) spectra keep their shape

dE/dt ~ E² (Synchrotron, IC)

- → relative losses stronger for high-energy particles
- → spectra become steeper

Spectra & radiation cooling

SOVIET ASTRONOMY - AJ

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NONSTATIONARINESS OF SPECTRA OF YOUNG SOURCES OF NONTHERMAL RADIO EMISSION

N. S. Kardashev

P. K. Shternberg State Astronomical Institute Translated from Astronomicheskii Zhurnal, Vol. 39, No. 3, pp. 393-409, May-June, 1962 Original article submitted August 11, 1961 see also Longair High Energy Astrophysics

The effect of various modes of energy losses by relativistic electrons on the spectrum of sources of nonthermal cosmic radio-frequency emission is considered, as well as the effect of the systematic and stochastic acceleration of relativistic electrons. Energy spectra are found for various combinations of energy buildup and loss processes, and the time variation of the spectra is studied. The expected synchrotron radiation spectra are obtained, and the variation of these spectra during the evolution of radio sources is investigated.
Spectra & radiation cooling



continuous injection with index Γ : "cooling break" $\Gamma \rightarrow \Gamma+1$ where $\tau_y \sim 10^7/E_{TeV} B_{\mu G}^2 < \tau_{source}$



Figure 19.2. (a) A solution of the diffusion-loss equation for steady-state injection of electrons with a power-law energy spectrum $Q(E) \propto E^{-p}$ in the presence of energy losses of the form $dE/dt = -aE^2$. (b) The time evolution of a power-law energy distribution injected at t = 0 with no subsequent injection of electrons. In this case p > 2. (c) As in case (b), but with p < 2.

Spectra & (synch.) radiation cooling





RXJ 1713.7-3946



Cooling in synchrotron spectra from SNR



 $\log \nu$, Hz

Crab Nebula: measuring the B field



Crab Nebula: Target fields for IC



Evolution of pulsar wind nebula



Active Galactic Nuclei



AGN variability (PKS 2155-304)





Topics

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The receiving end: Detecting gamma rays





Gamma-ray absorption in matter



Radiation length (characteristic scale for pair conversion) is 37 g/cm^2

1 H/cm³ over 10 kpc is 0.05 g/cm^2

Don't worry for Galactic gamma rays ...

Gamma-ray absorption by radiation

gamma ray

mm

CMB, vis, ... M Threshold: $s \approx 4E_{\gamma}E_{ph} > (2m_e)^2$ $E_{\gamma}[TeV] > 1/E_{ph}[eV]$

→ Always in "KN" regime

Cross section drops with incr. s

→ Near-threshold regime dominates

TeV gamma rays in the Galaxy: target density ~1 eV/cm³ or ~1 photon/cm³ assume Thomson cross section $(6.10^{-25} \text{ cm}^2)$ as upper limit

→ Mpc range (10²⁴ cm)

'bit more precise

Aharonian, VHE Cosmic Gamma Radiation, World Scientific, p. 117





H.E.S.S., astro-ph/0607192

20 solar mass star, Radius $\sim 7.10^{11}$ cm, Luminosity $\sim 10^{39}$ ergs/s Orbit radius $\sim 2.10^{12}$ cm Energy density ρ of starlight?



$$L = 4\pi r^2 \rho c \quad \Rightarrow \quad \rho \approx \frac{2 \cdot 10^3}{r_{12}^2} \text{ erg/cm}^3 \approx \frac{10^{15}}{r_{12}^2} \text{ ph/cm}^3$$
$$\int \rho \, ds \approx 10^{27} \text{ph/cm}^2$$
$$\sigma_T \approx 6 \cdot 10^{-25} \text{ cm}^2; \qquad \sigma_T \int \rho \, ds \gg 1$$

All TeV gamma rays from back side or orbit pair-produce

Extragalactic propagation





Measurement of EBL density

from gamma-ray attenuation



less model-dependent limits:

Mazin & Raue astro-ph/0701694

